Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Pressure Distribution on Blades in Cascade Nozzle for Particulate Flow

W. TABAKOFF* AND M. F. HUSSEIN†
University of Cincinnati, Cincinnati, Ohio

Nomenclature

A = cross-sectional area of stream tube (ft²)
α = particle concentration
C_p = specific heat (Btu/lb°R)
d_p = particle mean diameter (ft)
g = gravity constant
J = mechanical equivalent of heat
k_g = coefficient of conductivity for the gas (Btu/hr ft°R)
P = gas particle suspension pressure
p = pressure of gas-only flow
Pr = Prandtl number
R_g = gas constant
Re = Reynolds number
ρ = the gas-only flow density (lb/ft³)
ρ_g = the gas density in gas particle flow (lb/ft³)
ρ_p = the particle density (lb/ft³)
$\bar{\rho}_p$ = solid particle material density (lb/ft³)
T = the gas-only flow temperature (°R)
T_g = the gas temperature in gas particle flow (°R)
u = gas-only flow velocity (fps)
u_g = gas velocity in gas particle flow (fps)
u_p = particle velocity (fps)
W = mass flow rate of gas particle mixture or mass flow rate of gas-only flow in a stream tube (lb/sec)

Superscripts

$'$ = nondimensional parameter

Subscripts

g = gas
p = particle
l = condition at starting point of the stream tube
0 = total conditions

Introduction

THE pressure distribution on the blade in cascade increases with the introduction of solid particles to the gas flow.[1,2] The forces on the blade are larger thus requiring a stronger blade design. In this work, a theoretical approach to estimate the pressure distribution on the blade surface for a particulate gas flow for incompressible and compressible cases was developed. Gas flow without particles past a cascade was considered first. It was assumed that an experimental pressure distribution on the blade surface is known for given gas flow and inlet conditions. It was assumed that two stream tubes exist in the flowfield around the blade; one at the suction side and one at the pressure side. The gas flow without particles was used to determine the nondimensional area of the stream tube as function of the given pressure distribution and inlet gas conditions of the nonparticulate gas flow. The governing equations for the particulate flow were formulated for incompressible and compressible cases. The resulting differential equations were solved numerically for the pressure distribution and the particulate flow properties. The different parameters in the governing equations were nondimensionalized with respect to values at a starting point 1, which is in the vicinity of the blade leading edge. These parameters are expressed as follows in Eq. (1)

$$T' = T/T_1 \quad p' = p/p_1 \quad u' = u/u_1 \quad \rho' = \rho/\rho_1 \quad P' = P/P_1$$
$$u'_g = u_g/u_{g_1} \quad T'_g = T_g/T_{g_1} \quad \rho'_g = \rho_g/\rho_{g_1} \quad u'_p = u_p/u_{g_1}$$
$$\rho'_p = \rho_p/\rho_{g_1} \quad T'_p = T_p/T_{g_1} \tag{1}$$

$$A' = A/A_1 = 1/u' \quad \text{(Incompressible case)} \tag{2}$$

$$A' = A/A_1 = 1/u'\rho' \quad \text{(Compressible case)} \tag{3}$$

The definition of the particle concentration α is expressed as

$$\alpha = W_p/(W_p + W_g) = W_p/W \tag{4}$$

Incompressible Flow

The continuity and momentum equations for gas-only flow are

$$(1/u)du/ds + (1/A)dA/ds = 0 \tag{5}$$

$$\rho u du/ds = -(gdp/ds) \tag{6}$$

Substituting Eq. (5) into Eq. (6) and putting the resulting equation in nondimensional form by using Eq. (1), one gets an expression for the rate of change of stream tube nondimensional cross-sectional area as a function of the rate of change of the nondimensional gas without particles flow pressure.

$$dA'/ds = (A'/u'^2)(gp_1/\rho u_1^2)(dp'/ds) \tag{7}$$

where u' is given by

$$u' = [(p_0 - p/p_0 - p_1)]^{1/2} \tag{8}$$

The equation for the drag force on the solid particles, the momentum equation of gas and particles, and the continuity equation of gas are

$$u_p du_p/ds = 18\mu_g/\bar{\rho}_p d_p^2(u_g - u_p) \tag{9}$$

$$dP/ds = -(W/gA)[(1-\alpha)du_g/ds + \alpha du_p/ds] \tag{10}$$

$$\rho_g u_g A = (1-\alpha)W \tag{11}$$

Differentiation of Eq. (11) and substitution of Eqs. (1) and (7) into the resulting differential equation results in

$$du'_g/ds = -(u'_g/A')(dA'/ds) = -(u'_g/u'^2)(gp_1/\rho u_1^2)(dp'/ds) \tag{12}$$

Substituting Eq. (1) into Eqs. (9) and (10), they take the nondimensional form

$$u'_p du'_p/ds = (18\mu_g/u_{g_1}\bar{\rho}_p d_p^2)(u'_g - u'_p) \tag{13}$$

$$dP'/ds = -\rho_g u_{g_1}^2/gP_1A'[du'_g/ds + (\alpha/1-\alpha)du'_p/ds] \tag{14}$$

Presented as Paper 71-82 at the 9th Aerospace Sciences Meeting, New York City, January 25–27, 1971, submitted February 1, 1971; revision received May 21, 1971. This work was sponsored under Contract DAHC04-69C-0016, U.S. Army Research Office—Durham,

Index Categories: Multiphase Flows; Subsonic and Supersonic Air Breathing Propulsion.

* Professor, Department of Aerospace Engineering. Associate Fellow AIAA.

† Graduate Research Assistant, Department of Aerospace Engineering.

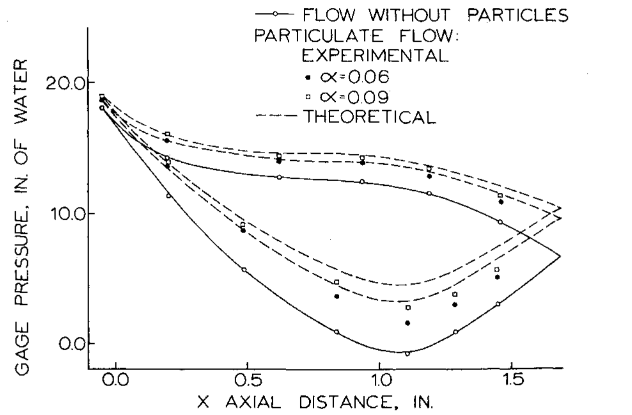

Fig. 1 Comparison between theoretical and experimental pressure distribution, incompressible flow.

A numerical solution of the flow governing Eqs. (12–14) yields the pressure distribution on the blade surface for a gas-particle flow for incompressible flow conditions.

Compressible Flow

For a gas passing through a stream tube the continuity, momentum, energy, and equation of state can be manipulated together with Eq. (1) to obtain the rate of change of the stream tube nondimensional area distribution as

$$dA'/ds = [R_g/JC_{pg}T' + (gp_1/\rho_1 u_1^2)1/u'^2 - 1/T'] \times (A'/\rho')dp'/ds \quad (15)$$

The energy equation of particulate flow can be written in nondimensional form using Eq. (1) as

$$\frac{dT'_g}{ds} = \frac{T'_g}{A'}\frac{dA'}{ds} - \frac{\alpha}{(1-\alpha)}\frac{\rho_g u_{g1}^2}{gP_1} u'_g \frac{du'_p}{ds} + u'_g \frac{du'_g}{ds}\left(\frac{T'_g}{u'^2_g} - \frac{\rho_{g1} u_{g1}^2}{gP_1}\right) \quad (16)$$

In nondimensional form, the equation of momentum of gas particle flow is

$$dP'/ds = -(\rho_{g1}u_{g1}^2/gP_1A')\{du'_g/ds + [\alpha/(1-\alpha)]du'_p/ds\} \quad (17)$$

The equation of state for the gas can be differentiated and written in nondimensional form using Eq. (1). Elimination of dp'/ds between the resulting equation and Eq. (17) results in the following expression

$$\frac{dT'_g}{ds} = \frac{T'_g}{A'}\frac{dA'}{ds} - \left(\frac{\alpha}{1-\alpha}\right)\frac{u_{g1}^2}{gR_gT_{g1}} u'_g \frac{du'_p}{ds} + \left(\frac{T'_g}{u'_g} - \frac{u_{g1}^2}{gR_gT_{g1}} u'_g\right)\frac{du'_g}{ds} \quad (18)$$

From Eqs. (16) and (18) one obtains

$$\frac{du'_g}{ds} = \left\{-\frac{T'_g}{A'}\frac{dA'}{ds} - \frac{\alpha}{(1-\alpha)}\left[\frac{C_{pp}}{C_{pg}}\frac{dT'_p}{ds} + \left(\frac{u_{g1}^2}{gJC_{pg}T_{g1}}u'_p - \frac{\rho_{g1}u_{g1}^2}{gP_1}u'_g\right)\frac{du'_p}{ds}\right]\right\} \Bigg/ \left\{u'_g\left[\frac{u_{g1}^2}{gJC_{pg}T_{g1}} + \frac{T'_g}{u'_g} - \frac{\rho_{g1}u_{g1}^2}{gP_1}\right]\right\} \quad (19)$$

The equation for the heat transfer between solid sphere particles and the gas flow can be derived in the nondimensional form as follows

$$u'_p\frac{dT'_p}{ds} = -\frac{6k_g}{\bar{\rho}_p C_{pp} d_p^2 u_{g1}}(T'_p - T'_g)(2 + 0.6Re^{0.5}Pr^{1/3}) \quad (20)$$

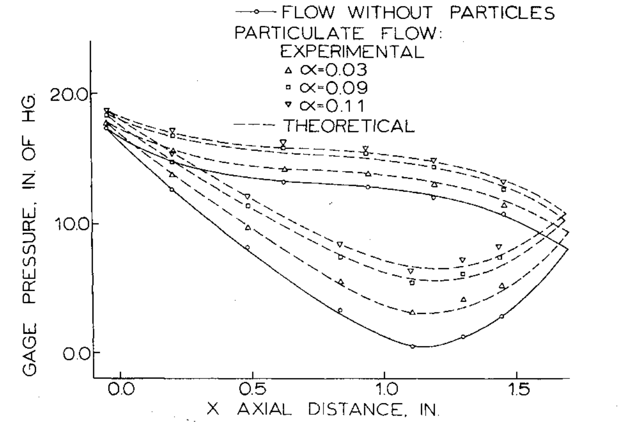

Fig. 2 Comparison between theoretical and experimental pressure distribution, compressible flow.

The equation of the drag exerted on the particles by the gas flow in nondimensional form was found to be

$$u'_p\frac{du'_p}{ds} = (u'_g - u'_p)\frac{18\mu_g}{u_{g1}\bar{\rho}_p d_p^2}\frac{28Re^{-0.85} + 0.48}{24Re^{-1.0}} \quad (21)$$

Equations (16, 17, and 19–21) are the gas particle suspension governing equations. These differential equations were solved numerically[2] for the value of the pressure of the particulate flow at the surface of the blade in the cascade nozzle for compressible flow cases.

Experimental Facilities

A special cascade tunnel[3,4] that provides a gas particle flow mixture was used in the experiments. A high-speed camera[5,6] was used to evaluate the particle's average initial velocity, $u_{\cdot p1}$

Comparison Between Experimental and Theoretical Pressure Distribution

The particles used were $\bar{\rho}_p = 68.673$ lb/ft³ and $d_p = 1000\ \mu$. Figure 1 shows the comparison between experimental and theoretical results for the incompressible flow case. The theoretical estimation gives fair agreement with the experimental data for the pressure distribution on the blade pressure side, while it slightly overestimates the value of the pressure near the rear half of the blade suction side. Figure 2 shows a good agreement between the experimental and theoretical results for the compressible flow case.

Figure 3 shows the effect of compressibility. It is noted that the deviation from the compressible case is smaller for the pressure side than for the suction side. Also, it is observed

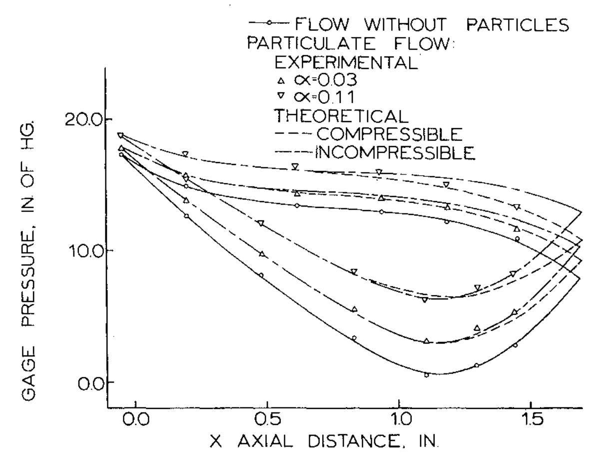

Fig. 3 Comparison between theoretical and experimental pressure distribution, compressibility effects.

that the deviation starts at the rear part of the airfoil where the gas velocity is higher.

Conclusion

The theoretical method developed here gives good agreement with experiments for different flow conditions in the incompressible and compressible flows. This theoretical method provides the designer with a means of estimating the forces on the blade in cascades. It has been observed that the compressibility effects are practically insignificant. This effect increases, however, with increasing α and is more pronounced at the rear part of the blade.

References

[1] Tabakoff, W. and Hussein, M. F., "Properties and Particle Trajectories of Gas Particle Flow," AIAA Paper 70-712, San Diego, Calif., 1970.

[2] Hussein, M. F. and Tabakoff, W., "The Properties of a Gas Particle Flow in Cascade," Project Themis Rept. 69-4, Sept. 1969, Univ. of Cincinnati, Cincinnati, Ohio; also U.S. Government Research and Development Rept. AD-697164.

[3] Tabakoff, W. and Hussein, M. F., "An Experimental Study of the Effect of Solid Particles on the Pressure at Blade Surface in Cascade," Project Themis Rept. 70-8, March 1970, Univ. of Cincinnati, Cincinnati, Ohio; also U.S. Government Research and Development Rept. AD-703896.

[4] Tabakoff, W. and Hussein, M. F., "Measurements of Particulated Gas Flow Pressure on Cascade Nozzles," *Journal of Aircraft*, Vol. 8, No. 2, Feb. 1971, pp. 121–123.

[5] Tabakoff, W. and Hussein, M. F., "Experimental Investigation of the Trajectories and Velocities of Solid Particles Entrained by Fluid Flows in Cascade Nozzle," Project Themis Rept. 70-12, August 1970, Univ. of Cincinnati, Cincinnati, Ohio; also U.S. Government Research and Development Rept. AD-711121.

[6] Tabakoff, W. and Hussein, M. F., "Trajectories of Particles Suspended in Fluid Flow Through Cascades," *Journal of Aircraft*, Vol. 8, No. 1, Jan. 1971, pp. 60–62.

Position of the Thrust Line and Longitudinal Stability

AMNON KATZ*
The University of Michigan, Ann Arbor, Mich.

Nomenclature†

A = aspect ratio
c = mean chord
C_D = $D/(qS)$ = drag coefficient
C_L = $L/(qS)$ = lift coefficient
C_M = $M/(qSc)$ = moment coefficient
C_{mu} = $(1/Sqc)(V\partial/\partial V)M_{c.g.} \approx -2C_D z_T$
$C_{m\alpha}$ = $(1/Sqc)(\partial/\partial\alpha)M_{c.g.} \approx s dC_L/d\alpha$
C_{xu} = $(1/Sq)(V\partial/\partial V)F_x \approx -2C_D$
$C_{x\alpha}$ = $(1/Sq)(\partial/\partial\alpha)F_x \approx C_L(1 - 2dC_L/d\alpha/\pi eA)$
C_{zu} = $(1/Sq)(V\partial/\partial V)F_z \approx -2C_L$
$C_{z\alpha}$ = $(1/Sq)(\partial/\partial\alpha)F_z \approx -dC_L/d_\alpha$
D = drag
e = efficiency factor in $C_D = C_{DO} + C_L{}^2/(\pi eA)$
F_x = $T - D$ = horizontal force
F_z = $mg - L$ = vertical force
g = acceleration in free fall
L = lift
M = aerodynamic moment
$M_{c.g.}$ = $M + Tz_Tc$ = over-all moment
m = mass of the airplane
q = $\frac{1}{2}\rho V^2$ = dynamic pressure
s = static margin (negative) expressed as a fraction of the chord
S = wing area
T = thrust
V = true airspeed
z_T = vertical displacement of the thrust line (positive downward) expressed as a fraction of the chord
α = angle of attack
γ = flight-path angle
δ = denotes a small variation in a quantity
ζ = damping ratio
η = L/D
ω = angular frequency

Received December 28, 1970; revision received June 17, 1971. The author takes pleasure in thanking D. L. Greenwood and R. M. Howe for introducing him to the field as well as for fruitful discussions.

* Visiting Research Associate, Department of Aerospace Engineering and Institute of Science and Technology.

† Time derivatives are denoted by dots over letters.

Introduction

THE phugoid motion is usually assumed to take place at a constant angle of attack.[1] The present Note points out that this is only true so long as the thrust line is not appreciably displaced from the airplane c.g. Expressions for the frequency and damping of the phugoid, which are valid when a displacement of the thrust line does occur, are presented.

The effect of a displaced thrust line on the frequency is easily appreciated qualitatively. Suppose the thrust line is below the c.g. In this case there exists a nose up thrust moment that is balanced in straight and level flight by a nose down aerodynamic moment. The two moments depend differently on airspeed, therefore a change of airspeed disturbs the balance. As the airplane noses down and picks up speed, the nose down aerodynamic moment overcomes the nose up thrust moment and tends to push the nose further down. This cuts down the "restoring force" and with it the frequency of the phugoid. This qualitative argument is borne out by the quantitative analysis below. The analysis shows that the damping of the phugoid is also reduced. It is found that a large displacement of the thrust line below the c.g. results in unstable phugoid oscillations and eventually in nonoscillatory divergence.

The analysis is carried out explicitly for a conventional low-speed airplane with the variation of thrust with airspeed neglected. Results of a more general analysis applicable to a general airplane are also presented.

Equations of Motion

The equations governing the phugoid motion are

$$mV\dot{\gamma} = L - mg\cos\gamma \tag{1}$$

$$m\dot{V} = T - D - mg\sin\gamma \tag{2}$$

$$0 = M + Tz_Tc \tag{3}$$

The last equation expresses the balancing of moments. It is assumed that a short period response of high frequency and damping makes sure that the moments are always very nearly balanced and angular accelerations may be neglected.

Consider a steady straight and level flight condition in which

$$\gamma = 0 \tag{4}$$

$$L = mg \tag{5}$$

$$D = T = \eta^{-1}mg \tag{6}$$

$$M = -Tz_Tc \tag{7}$$

When Eqs. (1-3) are linearized around this condition, one finds

$$mV\delta\dot{\gamma} = \delta L \tag{8}$$

$$m\delta\dot{V} = \delta T - \delta D - mg\delta\gamma \tag{9}$$

$$0 = \delta M + \delta Tz_Tc \tag{10}$$